**Chapter 2: Introduction to proof Test A** Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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|  | Identify the following numbers as either rational or irrational.  (a) −3  (b) 67.676 776 777 677 7…  (c)  (d) | (a) Rational [1 mark]  (b) Irrational [1 mark]  (e) Rational [1 mark]  (f) Irrational [1 mark] | 6 |
|  | Express the following common fractions as decimal fractions.  (a)  (b) | (a)  [1 mark]  (b)  [2 marks] | 3 |
|  | (a) Express  as a common fraction.  (b) Express  as a common fraction. | (a) Let .    [2.5 marks]  (b) Let .    [2.5 marks] | 5 |
|  | Determine if the following are true statements  (a) If *x* is a positive integer, then  is a positive integer.  (b) If , then .  (c) If today is Thursday, then yesterday was Friday.  (d) If  is rational, then *x* and *y* are integers. | (a) True [1 mark]  (b) True [1 mark]  (c) False [1 mark]  (d) False [1 mark] | 4 |
|  | For each of the following, write the converse and determine if implication () or equivalence () is the more appropriate symbol.  (a) If *x* is an even integer, then  is an even integer.  (b) If , then  (c) If *x* is odd, then 2*x* is even.  (d) If , then . | (a) *x* is even   is even  Converse:  is even  *x* is even  True  Therefore,  [1.5 marks]  (b)  Converse:    True  Therefore,  [1.5 marks]  (c) *x* is odd  2*x* is even  Converse:  2*x* is even  *x* is odd  False  Therefore,  [1.5 marks]  (d)  Converse:    False (2.5 + 0.5 = 3)  Therefore,  [1.5 marks] | 6 |
|  | Identify the negation of each of the following statements.  (a)  (b) The number is prime.  (c) 1 = 5  (d) The number is even. | (a)  [1 mark]  (b) The number is not prime. [1 mark]  (c)  [1 mark]  (d) The number is odd. [1 mark] | 4 |
|  | Identify the negation of each of the following statements.  (a) The numbers are even or prime.  (b) The numbers are multiples of 5 and 3.  (c) If *x* is even, then *x* + 1 is odd.  (d) If , then . | (a) The numbers are not (even or prime).  The numbers are odd and not prime.  [1 mark]  (b) The numbers are not (multiples of 5 and 3).  The numbers are not multiples of 5 or not multiples of 3.  [1 mark]  (c) *x* is even and *x* + 1 is even. [1 mark]  (d)  and  [1 mark] | 4 |
|  | Identify the contrapositive of the following statements  (a) If *x* is even, then *x* + 1 is odd.  (b) If *x* is odd, then  is odd.  (c) If it is raining, then today is Monday.  (d) If two angles are congruent, then they have the same measure.  (e) If a shape is a rectangle, then it has two pairs of congruent sides.  (f) If a shape is an equilateral triangle, it has three congruent sides. | (a) If *x* + 1 is even, then *x* is odd.  [1 mark]  (b) If  is even, then *x* is even.  [1 mark]  (c) If today is not Monday, then it is not raining.  [1 mark]  (d) If the two angles do not have the same measure, then they are not congruent.  [1 mark]  (e) If a shape does not have two pairs of congruent sides, then it is not a rectangle.  [1 mark]  (f) If a shape does not have 3 congruent sides, then it is not an equilateral triangle.  [1 mark] | 6 |
|  | Identify counter examples to disprove the following  (a) If *x* is a negative integer, then  is a negative integer.  (b) If *x* and *y* and integers, then  is an integer.  (c) If  is a positive integer, then *x* is a positive integer.  (d) If , then . | Sample responses:  (a)  [1 mark]  (b) ,  [1 mark]  (c)  [1 mark]  (d)  [1 mark] | 6 |
|  | Consider the statement ‘If *xy* is negative, then *x* is negative or *y* is negative.’  (a) Identify the contrapositive statement.  (b) Determine if the contrapositive statement is true.  (c) Determine if the original statement is true.  (d) Determine if the converse statement is true. | (a) Not (*xy* is negative) is .  Not (*x* is negative or *y* is negative) is  and .  Contrapositive:  If  and , then .  [2 marks]  (b) The statement is true.  [0.5 marks]  (c) As the contrapositive is true, the original statement is also true.  [0.5 marks]  (d) Converse:  If *x* is negative or *y* is negative, then *xy* is negative.  [1 mark]  This statement is false because it is possible for both *x* and *y* to be negative, and then *xy* is positive.  [1 mark] | 5 |

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|  | Prove that  is irrational. | R.T.P.  is irrational.  Proof by contradiction:  Assume  is rational. [0.5 marks]  [0.5 marks]  [1] [0.5 marks]  [0.5 marks]  Let , . [2] [0.5 marks]  Substitute [2] in [1]:    [0.5 marks]  This means that both *a* and *b* are divisible by 5.  [0.5 marks]  But . [0.5 marks]  Therefore, the assumption is false.  [0.5 marks]  Therefore,  is irrational. [0.5 marks] | 5 |

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|  | If a three-digit number is written down twice, to form a six-digit number, prove that the resulting number will have the numbers 7, 11 and 13 as factors. | Let the original number consist of the digits *a*, *b* and *c*. (  [0.5 marks]  The original number will be .  [1 mark]  The six-digit number will be    [2.5 marks]  Therefore, as 1001 is a factor of the six-digit number, 7, 11 and 13 will also be factors of the six-digit number.  [1 mark] | 5 |
|  | Prove that there are infinitely many rational numbers. | Assume that *x* is the largest rational number.  [0.5 marks]  As *x* is a rational number, , ,, .  [1 mark]      [1.5 marks]  As , ; therefore,  is a rational number.  [0.5 marks]  But *x* was the largest rational number.  Therefore, the assumption was false.  [1 mark]  There are an infinite number of rational numbers.  [0.5 marks] | 5 |
|  | Prove that it is impossible to find four different numbers *a*, *b*, *c* and *d* so that . | .  [0.5 marks]  Assume :    [3 marks]  Therefore,  or .  [1 mark]  Therefore, it is impossible to find four distinct integers to make the statement true.  [0.5 marks] | 5 |